RELATIONAL KNOWLEDGE AND SUCCESSFUL PROBLEM SOLVING IN ALGEBRA

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The quality of knowledge underlying students' algebraic thinking is a major concern of K-12 mathematics teaching and reform-driven curriculum documents. In this paper the author examines this issue by characterising a group of high school students' algebraic knowledge and patterns of use of that knowledge during the solution of selected problems. Results show that these students tended to show acceptable levels of proficiency with problems that involved substitution of values and simplification of equations. Students experienced difficulties with the interpretation of algebraic relations. Multiple regression analysis showed that knowledge about equations and pattern formation was a predictor of students' success at interpretation of graphs of linear functions. These results are interpreted as suggesting that understanding and analysis of graphs could be facilitated by teaching approaches that focus on the construction of robust schemas about patterns and symbolic relations in algebra.

INTRODUCTION

Algebra provides a conceptual foundation for the understanding of a range of other concepts that students are expected to learn in school mathematics. The importance of this area of mathematics has been underlined by the increasing attention the teaching and learning of algebra have received over the past decade from teachers and researchers alike. Children's understanding of algebraic concepts begin in the early years of their school life and continues throughout their mathematics learning experiences in high school and beyond.

The ubiquity of the subject matter of algebra in K-12 mathematics curriculum further attests to its critical role in helping students develop an appreciation of links that exists among other topics in mathematics. Indeed, this issue has been given considerable attention in the agenda of major curricular documents (National Council of Teachers of Mathematics, 1989, 2000). The recently concluded 12th ICMI Study on the theme, 'The Future of the Teaching and Learning of Algebra' further highlights the importance of algebra and brought into focus the many difficulties faced by students in learning algebra.

Despite significant strides that we have made in improving students' confidence and competence in using algebraic skills and concepts, it has been suggested that more work needs to be done in this area as students continue to experience difficulty in going beyond the meaningless manipulation of equations and symbols (Chazan, 1996; Stacey & MacGregor, 1999; Kirshner & Awtry, 2004). In the study reported here the author address this issue by exploring the nature of algebraic knowledge that drives students' cognition during problem solving.

THEORETICAL CONSIDERATIONS

Connections and mathematical understanding

The development of mathematical understanding has been analysed from a number of vantage points. Of these, the investigation of connections constructed by students has been a central theme of recent debate. It has been suggested that students learn mathematics best when they are encouraged to 'organise their information by making connections and forming relationships' (Sowder, 2001, p.4). Hence the analysis of connections seems to be an effective research strategy in the examination of mathematical understanding. The focus on connections has had a long history in psychological literature on concept development and problem solving not only in mathematics but also in other domains such as geometry, chess and physics. In their analysis, Carpenter and Lehrer (1999) characterised mathematical understanding as involving problem solving, constructing relationships and reflecting on one's own previous experiences with a particular topic of mathematics. As mathematical understanding is a developmental process, models of learning that specify constructions are appropriate for describing relations between the above activities. The quality of the connections can also be expected to have a major impact on how prior knowledge is used in a variety of learning situations (Schoenfeld, 1992).

Research from cognitive psychologists and mathematics educators has advanced several theoretical frameworks about concepts and their growth. In this paper, the author adopts the network perspective in making judgments about mathematics knowledge development. According to this view, conceptual growth and mathematical understanding can be interpreted in terms of the building of organised knowledge clusters called *schemas*. Schemas can be visualised as knowledge structures or chunks having one or more core concepts which are connected to other concepts and/or schemas by relational statements.

According to this framework of knowledge development, the quality of a schema is a function of two variables: the spread of the network and the strength of the links between the various components of information located within the network (Anderson, 2000). A complex schema can be characterised as having a large number of network of nodes that are built around one or more core concepts. Further, in a mature schema, the links between the various nodes in the network are robust, a feature which contributes to the ready accessing and use of that schema during problem-solving and other learning situations. A well-structured schema can also benefit students by helping them assimilate incoming new mathematical information with less cognitive effort.

The acquisition of mathematical concepts in K-12 can be seen as the construction of schemas each with differing levels of organization and complexity. The difference between a good student and a poor student is that the good student has built up schemas that are more complex, dense and better organised than his low-achieving peer. Chinnappan (1998) used the schema framework to compare the quality of geometry knowledge between high- and low-achieving students. Thus, a useful strategy would be to analyse the schemas of students for gaps in their knowledge and organisational quality. According to the schema framework of knowledge and performance, an impoverished schema is not conducive to solving novel problems and describing relations among concepts in mathematics because it does not help students extend their prior knowledge to new boundaries of understanding. Such schemas can be characterised as having a limited number of conceptual points to connect with.

STRUCTURE OF ALGEBRAIC SCHEMA

While schemas provide a broad theoretical framework for analysing organisational features of students' algebraic knowledge there is a need to disentangle components of the schema that underlies algebraic understanding. Literature on the development of algebraic understanding has advanced two constructs: procedural and conceptual structures. Broadly speaking, students who have attained procedural understanding can be expected to perform operations involving algebraic expressions such as simplifying equations. Conceptual understanding, on the other hand refers to the elucidation of relations between algebraic expressions and components that make up a particular algebraic statement. Sard (1991) used a *process-object* model to articulate the relationship between procedural and conceptual elements of algebra.

Students' experiences with algebra begin with the acquisition of knowledge about procedures or operations that are used in dealing with algebraic situations. These procedures include strategies and rules for simplifying, factoring and solving equations. Also included in this set of skills is an understanding of conventions and symbols that are used to represent algebraic expressions. One such convention could be the use of letters to represent variables or $f(x)$ to represent function of x. As students' experiences with algebra mature they are able to transfer knowledge of procedures to conceptual characteristics of relations. Kieren (1992) referred to this advancement as the evolution from the 'procedural to structural' (p.413). She argued that most students learn procedural skills but do not make the transition to structural understanding. Tall and Thomas (1991) also alluded to this link in their analysis of the nature of difficulties faced by students in learning algebra. Students who have developed multiple representations of an algebraic relationship can be expected to show high levels of structural understanding of variables that are embedded in that relationship.

The above analysis suggests that, among other things, algebraic schemas consist of networks of information nodes that are procedural and conceptual in nature. Accordingly, students who have built up a better connected and organised algebraic schema can be expected to make a smooth transition from the procedural to the conceptual aspects of the knowledge structure. For instance, in order for students to develop a sophisticated schema, say, about solution of quadratic equations, they need to make multiple connections among variables, families of equations and unknowns. As their schema become elaborated further one might expect to see them constructing equations to model a problem situation. In this sense the maturation of schemas can be seen as progressing along the procedural-conceptual continuum. Algebraic schemas with a higher proportion of procedural information can be argued to be less complex than one that has more conceptual information. In a problem situation both components of the knowledge base are necessary but conceptual knowledge is more useful in generating powerful representations of a given problem. In this sense, one could argue that the conceptual part of the schema is indicative of deeper understanding of algebra.

GRAPHICAL INFORMATION

Although schemas provide a useful conceptual framework within which to examine the growth of general algebraic thinking and knowledge development, the complexity of the area warrants that we decompose the macro-schema and consider the array of algebraic sub-schemas and their relations if we are to make progress in depicting the type of knowledge that could be useful for students to become proficient problem solvers in this domain. For example, equations and the solution of equations is one important part of students' knowledge. This knowledge could be organised into Equation Schema (ES). ES could include information that is conceptual as well as procedural. Pattern identification is an important skill in algebra (Kieren, 1992). The knowledge that drives this activity could reside in the Pattern Schema (PS). PS could include a range of information including strategies for exploring patterns in a given situation. While ES and PS appear to be two distinct entities, there are important relations between the two knowledge structures. For instance, the fact that some patterns can be expressed as equations is an expression of relational information between the schemas. A third area in algebra involves graphs and graphing. This area of the learner's algebraic knowledge could be located in Graphical Schema (GS). Skills in drawing and exploring the meaning of variables that are anchored by graphs are closely linked to students' understanding of patterns and equations. This line reasoning suggests that the elucidation of connections among these schemas would help us gain further insight into the growth of algebraic understanding. In the present study, this issue is addressed by investigating knowledge-based predictors of successful performance in graphical problems.

The purpose of the present study was to describe the quality of algebraic schema developed by a group of Year 10 students. In particular, the author was interested to examine the type of procedural and conceptual knowledge that students access in problem contexts, and the integration of these knowledge components during graphical representation of equations. The research questions for this study are:

- What is the nature of procedural knowledge that Year 10 students activate during the solution of algebra problems?
- What is the nature of conceptual knowledge that Year 10 students activate during the solution of algebra problems?
- Is there evidence of transition from using procedural to conceptual knowledge during the solution of algebra problems among Year 10 students?
- Are there relations among algebraic schemas that have differential procedural and conceptual information?
- What are the indicators of success in graphical representation of linear equations?

METHODOLOGY

Participants

The participants in this study consisted of 58 Year 10 students (28 males and 30 females) from a metropolitan high school. The students came from two classes representing 'average to above average' ability levels of the school's Year 10 cohort. All students had completed the algebra topic. Students in the study also reflected the socio-cultural composition of the local community.

Instrument

The *Algebra Schema and Access Instrument* (ASAI) was developed for the study. The instrument contained 16 problems all of which required the accessing and use of algebraic knowledge. Problems 14 and 15 consisted of three (14a, 14b, 14c) and two parts (15a, 15b) respectively. An important consideration in the development of the instrument was the identification of algebraic schemas that one would expect Year 10 students to activate during the course of their solution attempts. It is important to point out that I have used a problem-based schema identification approach, and that schemas activated by the students were necessarily limited by the problem contexts. Selected items from ASAI appear in Appendix 1.

It is possible that a non-problem-based strategy could be expected to generate a different set of schemas. However, a problem-driven schema activation and use by the students could be argued for to provide a more complete picture about the quality of schemas that students have built up because it has the potential to reveal more complex connections among procedural and conceptual components that exist not only within schemas but among schemas. The latter complex of connections among schemas has been argued to exert a major influence in the construction of problem representations (Sweller & Cooper, 1985). This line of reasoning was used in classifying the 16 problems into six Problem Categories (PCs) as shown in Table 1. A selected set of the problems from each category is provided in Appendix 1.

Table1
Problems and problem schema categories

Problems	Problem Category (PC)
1 and 2	A - Factorisation
3 and 4	B - Evaluation
5, 6, 7, 8, 9 and 10	C - Solution of Equations
11, 12 and 13	D - Word Problems
14a, 14b and 14c	E - Pattern Generation
15a, 15b and 16	F - Graphical Interpretation

Procedure

The class teachers administered the instrument to students during normal class periods. The study was conducted in the fourth term of the school's academic year. Students were given 60-90 minutes to complete the problems. Students were encouraged to attempt all the 16 problems. They were also asked to write down every step in their solution attempts even if they did not arrive at the 'correct' answer. Students were permitted to use calculators if required.

The following scoring scheme was developed to code students' solution attempts. There were two major considerations in this scheme: solution approach and generation of relevant values. The former was concerned with problem representation and the latter factor provided information on the use of appropriate schemas to generate values that were relevant to the problem representation. While solution outcome was important it was not the sole factor in the scoring scheme. The approach-based analysis is argued to provide a measure of the type of schemas that were used by the students during the search for solution. The scheme was trialled with two independent coders who were mathematics teachers and researchers in order to resolve potential differences in the interpretation of the codes. The final scoring system was used to code students' solution transcripts.

0 - No attempt was made to solve the problem.
1 - Solution was attempted but both the approach and values generated were incorrect.
2 - Solution was attempted with a correct approach but none of the values generated were correct or relevant; incorrect solution outcome.
3 - Solution was attempted with a correct approach; one correct value was generated; incorrect solution outcome.
4 - Solution was attempted with a correct approach; two correct values were generated; incorrect solution outcome.
5 - Solution was attempted with a correct approach; three or more correct values were generated; correct solution outcome.

Results

Results of the analysis of students' solution attempts to individual problems are presented in Table 2. The results show that students attempted all the 16 problems with varying degrees of success. The means and standard deviations indicate that some problems were more difficult (Problems 6-10, 13-16) than others (Problems 1, 2, 3, 11 and 12). From the representational angle the solution attempts reveal a number of patterns. Firstly, students constructed correct representations for most of the problems in PC A and PC B, and a few problems in PC C. In PC D, students experienced more success with problems 11 and 12 in comparison with 13. In general, students experienced difficulty with the solution of all the problems in PCs E (Pattern Generation) and F (Graphical Representation). In more than 50% of the problems presented students scored mean values of 1 or less indicating failure to attempt or construction of incorrect problem representations.

Results in Table 2 also provide insight into the approach the students adopted in tackling the problems. A mean value that is less than 2 can be interpreted as suggesting that the participating students either did not attempt a particular problem or used an incorrect approach in their attempt to reach the problem goal. Table 2 shows that this was evident in the case of problems 6, 7, 8, 9b and 10 (PC C). While students had experienced success with some of the problems in this category, with the more difficult problems, students tended either not attempt or adopt an incorrect solution strategy. This pattern in their problem solving behaviour was also evident in the other PCs: Problem 13 (PC D); problems 14a, 14b, 14c (PC E), problems 15a, 15b, 16 (PC F).

Table 2
Descriptive Statistics of Solution Scores

Problems	Mean	SD
1	2.55	1.59
2	3.59	1.60
3	4.64	1.18
4	2.19	2.08
5	2.12	1.76
6	1.69	1.85
7	1.71	1.36
8	1.48	1.53
9a	2.17	2.15
9b	0.66	1.19
10	1.09	1.42
11	3.48	1.75
12	4.48	1.41
13	0.93	0.72
14a	2.40	2.25
14b	1.64	2.13
14c	1.21	1.87
15a	1.48	1.75
15b	1.21	1.63
16	1.02	1.10

Correlations among representations

In order to examine the potential relations among the five problem categories and associated schemas, Pearson correlation coefficients were computed. The results of this analysis appear in Table 3. There were low correlations between the Pattern Generation and the other five PCs. The correlations among Factorisation, Evaluation of Solutions, and Graphical Interpretation PCs can only be described as moderate. Scores on Word Problems PC had little or negligible association with the other PCs. The activation of schemas for the Factorisation PC was moderately correlated with those relevant to Solution of Equations and Graphical Interpretation.

Table 3
Correlations among problem categories

	Factorisation	Evaluation	Solution of Equations	Word Problems	Pattern Generation	Graphical Interpretation
Factorisation		0.51*	.60**	.09	.28*	.53**
Evaluation			.43**	.17	.23*	.45**
Solution of Equations				.15	.31*	.58*
Word Problems					.05	.26*
Pattern Generation						.46**

**p<.01; *p<0.05

Regression analysis

The link knowledge about graphs, equations and pattern was the focus of the final research question. Specifically, the interest here was to examine the predictability of activation of Graphical Schema (GS). In order to answer this question, a standard multiple regression was performed between GS as the dependent variable and Equation

Schema (ES) and Pattern Schema (PS) as independent variables. Hair, Anderson, Tatham and Black (1995) argued that the minimum R^2 that can be found with a Power of .80 (a = 0.05) for a sample of 50 cases involving 2 independent variables was 19. The authors also indicated that the desired ratio of number of independent variables to sample size for the purposes of generalizability of results of a regression analysis is about 1 to 15-20. The present study involved the participation of 58 students and two independent variables. This ratio of participants to independent variables is, therefore, regarded as being sensitive to R^2 values that are above 19, as well as meeting the requirements for generalizability of results.

As the regression analysis was concerned with linear relations and associated schemas, it was necessary to exclude any items from ASAI that involved quadratic equations. GS was measured on the basis of scores on items 15a and 15b. Scores for ES was based on the total solution scores of items 5, 6, 9 and 10. The second independent variable (PS), likewise, was computed by summing up scores on items 14a, 14b and 14c.

Table 4 shows the bi-variate correlations among the three variables. There were significant positive correlations amongst the dependent and independent variables. There was evidence of moderate correlation between the two independent variables.

Table 4
Bi-variate Correlations

	GS	ES	PS
GS	1.000	.678**	.460**
ES		1.000	.316*
PS		1.000	

** $p < 0.01$; ** $p < 0.05$

A stepwise regression analysis was performed using SPSS REGRESSION in order to determine if addition of scores for ES and PS improved prediction of scores on GS. Table 5 shows the model summary. Table 6 and 7 show associated ANOVA and Coefficients respectively. As can be seen from Table 5, activation of ES was the best indicator of success at solving graphical problem as measured by ES ($F(1,56) = 47.7$, $p<0.001$)) accounting for 45% of the variance (adjusted R^2). The addition of PS accounted for additional 6% of the variance, yielding a significant two variable model, $F(2, 55) = 30.6$, $p<0.001$. Overall, GS, which involved the activation of schemas relevant to analysis of graphs, was best predicted by activation of knowledge associated with equations and pattern identification.

Table 5
Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.678[a]	.460	.450	2.3288
2	.726[b]	.527	.510	2.1992

a Predictors: (Constant), ES
b Predictors: (Constant), ES, PS

Table 6
ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	258.707	1	258.707	47.703	.000[a]
	Residual	303.707	56	5.423		
	Total	562.414	57			
2	Regression	296.414	2	148.207	30.644	.000[b]
	Residual	266.000	55	4.836		
	Total	562.414	57			

a Predictors: (Constant), ES
b Predictors: (Constant), ES, PS
c Dependent Variable: GS

Table 7
Coefficients

		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
Model		B	Std. Error	Beta		
1	(Constant)	-.356	.537		-.664	.509
	ES	.394	.057	.678	6.907	.000
2	(Constant)	-.794	.531		-1.497	.140
	ES	.344	.057	.592	6.056	.000
	PS	.158	.056	.273	2.792	.007

Dependent Variable: GS

DISCUSSION

The present study attempted to answer five questions that are related to high school students' knowledge and understanding of algebraic concepts by examining schema activation during problem solving. An algebraic schema can be constructed from an array of information. In the present study, the focus was on the composition of the schemas and their relations.

While schemas vary in their content and complexity of connections, it was argued that the solution of problems in PCs A and B would demand information that was more procedural in character than those in other PCs. Analysis of solution attempts to problems that mainly involved application of procedural skills (PCs A and B) indicated that students had acquired a reasonable level of this type of knowledge of algebra. This was evidenced by the high proportion of success with problems that required substitution of numerical values into a given expression. Students also showed that they could expand and simplify algebraic expressions. However, students tended to experience difficulty in finding solutions to a number of equations that had a complex structure. Taken together, these results suggest that while students have built up a repertoire of process skills there were also knowledge gaps in their procedural knowledge. It is also possible that the solution of more complex equations in PC C required understandings that were supported by schemas that were more conceptually loaded.

Analysis of solution attempts relevant to research questions two and three focused on students' conceptual understanding of algebraic expressions and evidence of establishing links between conceptual and procedural elements in the given problems. The mean scores indicate a high proportion of the students could not generate equations to model a given situation, and solve that equation. In addition, students experienced difficulty in representing and interpreting graphical forms of given equations (Category F). An equation is a symbolic form of a relationship that can be expressed in a graphical mode. A large number of students who participated in this study failed to translate the symbolic to the graphical form of a given equation and vice versa. The present results are consistent with findings of a number of other recent investigations of problem solving that involves modelling of problems in terms of algebraic expressions (Nathan, Kintch, & Young, 1992; Schoenfeld & Arcavi, 1988).

In the present study, students performed poorly in a task that involved the establishment of a relation between two variables (Problems 14a, 14b, 14c). This problem can also be seen as exploring a pattern that exists between two sets of numbers. While a number of students could determine the value of one variable given the other, these students did not describe the overall relationship in any meaningful manner. All three problems were related in that 14a was a particular case of 14b and 14c. It is noteworthy that students made better progress with 14a than the other two problems. The solution to 14a could be used in deriving solutions to 14b and 14c by finding values of P for different value of Q. This approach would lead them to conclude that values of P are increasing in threes. Further reasoning along these lines would help them construct the general relation, P=3Q-2. The relative success with 14a shows that students were more comfortable with substituting values rather than visualising the pattern.

The above analysis reveals an important interaction between procedural and conceptual knowledge where the procedural schema appear to drive the actions of the students. I argue that the accessing of conceptual schema about relations and variables would aid in making progress in completing problems 14b and 14c. These results also reflect those obtained by Stacey (1989) who found that students had difficulty in reasoning that led to generalising a pattern among variables, and the lack of transition from procedural to conceptual knowledge of algebra (Kieren, 1992).

Students in the present study also did not seem to understand the notion of ordered pairs (x,y), and that there was a rule that connected values x with values of y. This misunderstanding was evident in the solution attempt of Problem 10 where students were required to decide if a point was a solution to the given equation. It would seem that these students had yet to establish a link between the coordinates of the point and the rule that was expressed in the equation. The mean score for this problem was 1.09 suggesting most students did not attempt or used an incorrect representation. Solution attempts to Problem 10 again provide further support to the claim that students' algebra schema lacked the appropriate conceptual information.

The fourth question in the present study focussed on potential relations that might exist among algebraic schemas with different proportions of procedural and conceptual information. Correlation analysis showed problem categories that required the use of procedural information were moderately associated with each other such as Factorisation and Evaluation of algebraic expressions. As expected there were weak or absence of relations among problem categories that required the use of a blend of procedural and conceptual information. For example, solution to Factorisation problems was not associated strongly with solution to problems in the Word and Pattern Generation category (Table 3). These results suggest that students either had not developed the required conceptual schemas or tended not to integrate their procedural knowledge with conceptually organised schemas. The moderate positive association amongst schemas from the categories of Graphical Interpretation, Factorisation and Solution of Equations was interesting perhaps suggesting the role of procedural schemas in analysing linear functions.

Some tentative implications for skill development in the understanding of graphs and graphing can be identified from the data. Regression analysis suggests that the activation of equation and pattern generating schemas best predict students' performances in problems that involved interpretation of graphs. The ability to recognize the structure of equations and relations embedded among the variables constitutes a key factor in the visual analysis and representation of graphs. The predictor variables can be seen as a blend of items of information that would assist students in making connections between the symbolic and graphical models of linear relationships. The results reported here are also compatible with Leinhardt, Zaslavsky and Stein's (1990) view that algebraic and graphical data are separate but related in powerful ways. This study, along with that of Bell and Janvier (1981) reinforces the notion that the interpretation of graphs is predicated on multiple schemas, and that the assessment of understanding in this area of school mathematics should involve an array of assessment techniques.

On a more general level, the results of the present study are relevant to the debate over the causal relations between procedural and conceptual knowledge not only in the solution of algebra problems but mathematical problems. The present result is consistent with earlier research by Rittle-Johnson and Alibali (1999) who found that conceptual knowledge has a greater influence not only on the understanding of problems but also on the further development of procedural knowledge.

While it is too early to speculate, the present findings do suggest that teaching needs to focus on the development of both procedural and structural or conceptual aspects of algebra. It seems that higher levels of procedural skill development is a necessary but not a sufficient condition for students to solve problems that involve generation and manipulation of variables in an equation. Thus learning experiences need to make explicit the connections between these two aspects of algebraic knowledge.

This study represents a modest attempt at exploring the construction of representations for algebraic problems and the nature of schemas that support that construction. While there is some support here for the claim that teaching and learning algebra needs to focus on facilitating the building of more conceptually based schemas there is a need for a fine-grained analysis of algebraic schemas that drive problem representation.

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APPENDIX 1

1. Factorise the expression $3x^{2} + 6x - 9$

4. If $f(x) = 3x^2 - 7x$, what is the value of $f(2.5)$?

7. Solve the equation $x + \frac{6}{x} = 5$

11. A photograph is 3 cm longer that it is wide. Its area is 40 cm^2.
Find its length and width.

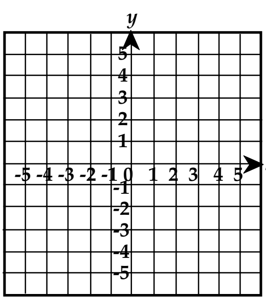
Q	*P*
1	1
3	?
4	10
6	16
n	?

14(a). What is *P* when *Q* = 3?

14(b). What is *P* when *Q* = *n*?

14(c). Describe the relationship between *P* amd *Q*.

15(a). Graph the equation $5y = -15 + 3x$ using the following grid.



15(b). Graph the equation $y = \frac{7}{2}$ using the following grid.